UDK 004.9

Multifractal detrended fluctuation analysis in Octave for ECG data

D.V.Belkov, E.N.Edemskaya, L.A. Lazebnaya, V.N. Boyko
Donetsk National Technical University
e-mail: belkovdv@list.ru

Abstract
Measuring the electrical impulses of the heart through an electrocardiogram (ECG) has become the primary method for detecting problematic heart conditions such as heart attacks or cardiac arrhythmia. Gaining more insight into the dynamical behavior of heartbeat irregularities would have meaningful applications in cardiology, especially if an "irregular" heartbeat could be characterized as being monofractal or multifractal, as opposed to being merely random. Using times series from electrocardiogram data, anyone is able to apply multifractal detrended fluctuation analysis (MDFA) to investigate possible multifractal structure of ECG data. Our work is aimed to test multifractal structure in time series data from an electrocardiogram. We examine data from ECGControl program. The research is executed in Octave runtime.

Introduction

The structural characteristics of biomedical signals are often visually apparent, but not captured by conventional measures like the average amplitude of the signal. Biomedical signals from a wide range of physiological phenomena possess a scale invariant structure. A biomedical signal has a scale invariant structure when the structure repeats itself on subintervals of the signal.Formally, the biomedical signal $X(t)$ are scale invariant when $X(ct) = c^H X(t)$. Fractal analyses estimates the power law exponent, H, that defines the particular kind of scale invariant structure of the biomedical signal. Fractal analyses are frequently employed in biomedical signal processing to define the scale invariant structure in ECG.Several reports during the last decade suggest that changes in the scale invariant structure of biomedical signals reflect changes in the adaptability of physiological processes and successful treatment of pathological conditions might change fractal structure and improve health. Fractal analyses are therefore promising prognostic and diagnostic tools in biomedical signal processing.

Monofractal and multifractal structures of the biomedical signal are particular kind of scale invariant structures. Most commonly, the monofractal structure of biomedical signals are defined by a single power law exponent and assumes that the scale invariance is independent on time and space. However, spatial and temporal variation in scale invariant structure of the biomedical signal often appears. These spatial and temporal variations indicate a multifractal structure of the biomedical signal that is defined by a multifractal spectrum of power law exponents. As an example, age related changes in the scale invariant structure of heart rate variability are indicated by changes of the multifractal spectrum rather than a single power law exponent.The width and shape of the multifractal spectrum can also differentiate between the heart rate variability from patients with heart diseases like ventricular tachycardia, ventricular fibrillation and congestive heart failure [1].

The aim of the work is to test multifractal structure in time series data from an electrocardiogram. The task is to examine data from ECGControl program [2, 3].The research is executed in Octave runtime.

Description of data

The electrocardiograph has information about electrical potentials on the surface of the skin. To do this, the electrodes attached to the chest and extremities were applied. These electrodes are called leads. Up to 6 leads are typically set on the chest and extremities. Chest leads V1-V6, and denotes the leads on the limbs are called main (I, II, III) and reinforced (aVR, aVL, aVF). All the leads give a different picture of the fluctuations, but summarizing information from all the electrodes, you can find out details of the heart as a whole.

Computer console ECG Light together with the program ECGControl allows you to record the electrocardiogram in six standard leads, save as a file, print, and carry out ECG contour analysis. In the area of leads the fragment window displays the period averaged electrocardiograms, on the selected slice three standard (I, II, III) and three enhanced leads (aVR, aVL, aVF).

Having a total picture of these elements ECG you can judge the EHA (electrical heart axis), which

indicates the presence of blockades and helps determine the location of the heart in the chest.There is a special coordinate system, which is located on the chest and рудзы to determine which side has an axis. Electrodes ECG, respectively, are located in this coordinate system. In a healthy person the right ventricle somewhat more left, so on the ECG heart axis deviates slightly to the right and down. It is considered to be the norm. In this case, the cardiogram is called the normogram [4]. We examine the normogram from ECGControl program. Figure 1, 2 show time series for three standard (I, II, III) and three enhanced leads (aVR, aVL, aVF).

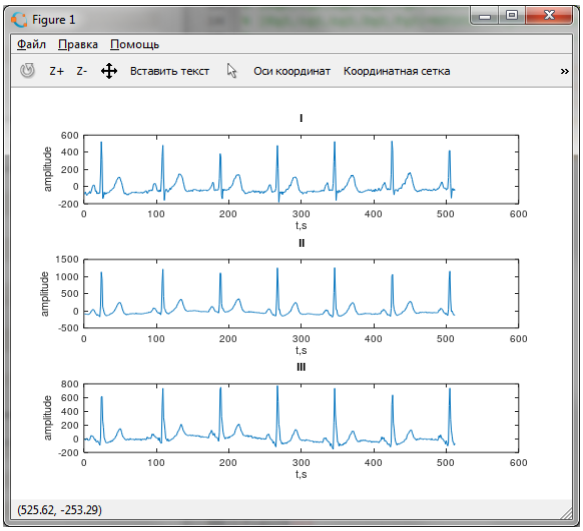

Figure 1 - Time series for standard (I, II, III) leads

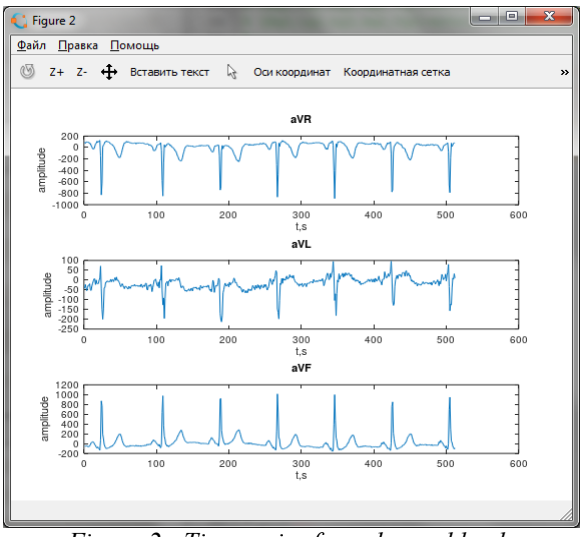

Figure 2 - Time series for enhanced leads

Description of MDFA

The structure of the monofractal and multifractal time series are different even though they have similar overall root-mean-square (RMS) and slopes H. The multifractal time series have local fluctuations with both extreme small and large magnitudes that is absent in the monofractal time series. The absence of fluctuations with extreme large and small magnitudes results in a normal distribution for the monofractal time series where the variation is described by the second order statistical moment (i.e., variance) alone. Monofractal detrended fluctuation analysis (DFA) is therefore based on the second order statistics of the overall RMS.

In the multifractal time series, local fluctuation, RMS(v), will be extreme large magnitude for segments v within the time periods of large fluctuations and extreme small magnitude for segments v within the time periods of small fluctuations. Consequently, the multifractal time series are not normal distributed and all q-order statistical moments should to be considered. Thus, it's necessary to extend the overall RMS in the monofractal DFA to the following q-order RMS of the multifractal DFA.The important steps involved in this method of analysis are mentioned here:

Step1: Computing the average

Let us suppose $x(i)$ for $i= 1 \ldots N$, be a non-stationary time series of length N. The mean of the above series is given by

$$x_{ave} = 1/N \sum_{i=1}^{N} x(i) \qquad (1)$$

Step 2: Computing the integrated time series

$$Y(i) = \sum_{k=1}^{t} (x(k) - x_{ave}) \text{ for } i=1,\ldots N. \qquad (2)$$

Step 3: Dividing the integrated time series to N_s non-overlappingbins (where $N_s = \text{int}(N/s)$ and s is the length of the bin) and computing the fluctuation function. Since N is not a multiple of s, so in order to include this part of the series the entire process is repeated starting from the opposite end. Thus, $2N_s$ bins are obtained and for each bin we perform least square fit of the series and then determine the variancefor each bin $\nu = 1,2,\ldots N_s$:

$F^2(s,\nu) = 1/s \sum_{i=1}^{s} (Y((\nu-1)s+i) - y_\nu(i))^2$,determine the variance for each bin $\nu = N_s+1,\ldots 2N_s$:

$F^2(s,\nu) = 1/s \sum_{i=1}^{s} (Y((\nu-N_s)s+i) - y_\nu(i))^2$, $y_\nu(i)$ is the least square fitted value in the bin ν .

Step4: Computing fluctuation function

The q-th order fluctuation function $F_q(s)$ is obtained after averaging over $2N_s$ bins.

$$F_q(s) = 1/(2N_s) \sum_{\nu=1}^{2N_s} (F^2(s,\nu))^{q/2})^{1/q} \qquad (3)$$

where q is an index which can take all possible values except zero because in that case the factor 1/q blow s up. F_q cannot be obtained by the nor m al aver aging procedure; instead a logarithmic aver aging procedure is applied

$$F_0(s) = \exp(1/(4N_s) \sum_{\nu=1}^{2N_s} \ln(F^2(s,\nu)) \sim s^{h(0)} \qquad (4)$$

Step 5: The procedure is repeated by varying the value of s. $F_q(s)$ increases with increase in value of s. If the series is long range power correlated, then $F_q(s)$ will show power law behavior $F_q(s) \sim s^{h(q)}$.

If such a scaling exists $\ln(F_q(s))$ will depend linearly on ln(s) ,with h(q) as the slope. In general the exponent h(q) depends on q. For stationary time series h(2) is identical with the Hurst exponent H. h(q) is said to be the generalized Hurst exponent. A monofractal time series is characterized by uniqueh(q) for all values of q.

The generalized Hurst exponent h(q) of MF-DFA is related to
the classical scaling exponent τ(q) by the relation

$$\tau(q) = qh(q) - 1 \qquad (5)$$

A monofractal series with long range correlation is characterized by linearly dependent q order exponent $\tau(q)$ with a single Hurst exponent H. Multifractal signal have multiple Hurst exponent and $\tau(q)$ depends non-linearly on q. The singularity spectrum $f(\alpha)$ is related to *h(q)* by

$$\alpha = h(q) + qh'(q) \qquad (6)$$

$$f(\alpha) = q(\alpha - h(q)) + 1 \qquad (7)$$

where α is the singularity strength and $f(\alpha)$ specifies the dimension of subset series that is characterized by α. The multifractal spectrum is capable of providing information about relative importance of various fractal exponents in the series e.g., the width of the spectrum denotes range of exponents. A quantitative characterization of the spectra may be obtained by least square fitting it to a quadratic function around the position of maximum α_0.

$$f(\alpha) = A(\alpha - \alpha_0)^2 + B(\alpha - \alpha_0) + C \qquad (8)$$

where C is an additive constant $C = f(\alpha_0) = 1$. B indicates the asymmetry of the spectrum. It is zero for asymmetric spectrum.

The width of the spectrum can be obtained by extrapolating the fitted curve to zero. Width W is defined as

$$W = \alpha_1 - \alpha_2 \qquad (9)$$

with $f(\alpha_1) = f(\alpha_2) = 0$. It has been proposed by some groups that the width of the multifractal spectra is a measure of degree of multifractality. For a monofractalseries, h(q) is independent of q. Hence from relation (6) and(7) it follows that the width of the spectrum will be zero for amonofractal series. The more the width, the more multifractal isthe spectrum.

The origin of multifractality in a time series can be determined. Two basic sources of multifractality in the time series are: 1) multifractality due to broad probability density function for the values of the time series; 2) multifractality due to different long range correlations of the small and large fluctuations.

The origin of multifractality can be ascertained by analyzing the corresponding randomly shuffled series. In the shuffling procedure, the values are put into random order and hence allcorrelations are destroyed. Hence, if the multifractality is due tolong-range correlations, then the shuffled series exhibits a non-fractal scaling. On the other hand, if the originalh(q) dependencedoes not change, i.e., $h(q) = h_{shuffled}(q)$, then the multifractality is due to the broad probability density, which is not affected in theshuffling procedure. If both kinds of multifractality are present ina given series, the shuffled series will show weaker multifractalitythan the original series.

The autocorrelation exponent γ can be estimated from the relation given below:

$$\gamma = 2 - 2h(2) \qquad (10)$$

For uncorrelated or short-range correlated data, h(2) is expected to have a value 0.5 while a value greater than 0.5 is expected forlong-range correlations. Therefore, for uncorrelated data, γ has avalue 1 and the lower the value the more correlated is the data.

The basic component of MFDFA is the local fluctuation, RMS. Statistical parameters other than RMS can be used to define the local fluctuation in a time series. The MFDFA has been shown to perform as well as or better than these multifractal analyses. However, extensions of detrending procedure in MFDFA should be considered when the biomedical time series contains strong oscillatory or ramp-like trends [5-7].

Description of results

Figures3-5 show scaling function Fq (q -order RMS) and corresponding regression line computed by MFDFA for three standard (I, II, III) leads.

Figures 6-8 show scaling function Fq and corresponding regression line for three enhanced leads (aVR, aVL, aVF). The scaling functions Fq(dots) and corresponding regression slopes Hq (dashed lines) are q-dependent. It means that time series are multifractal. They are time series: norm_1, norm_2, norm_3, norm_avr, norm_avl, norm_avf.

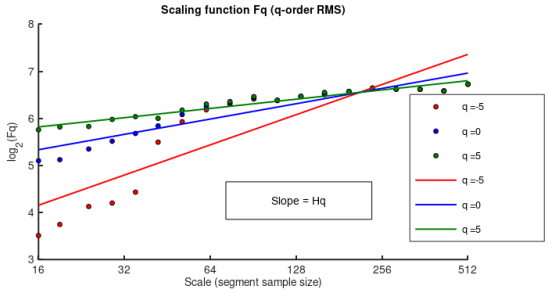

Figure 3- Function Fq for the norm_1

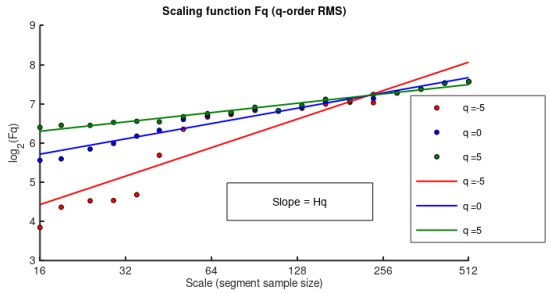

Figure 4- Function Fq for the norm_2

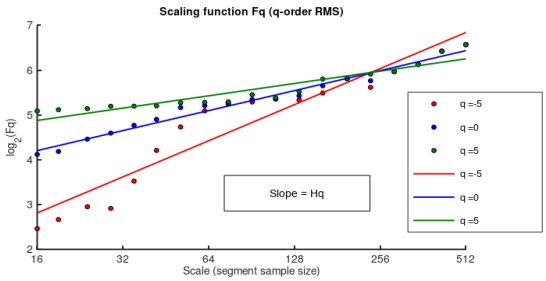

Figure 5- Function Fq for the norm_3

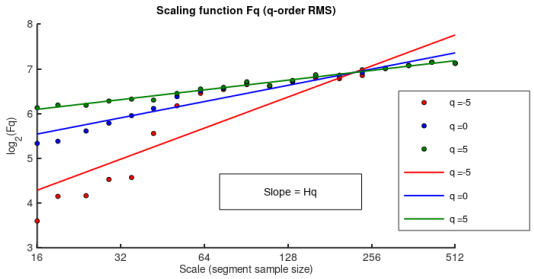

Figure 6 - Function Fq for the norm_avr

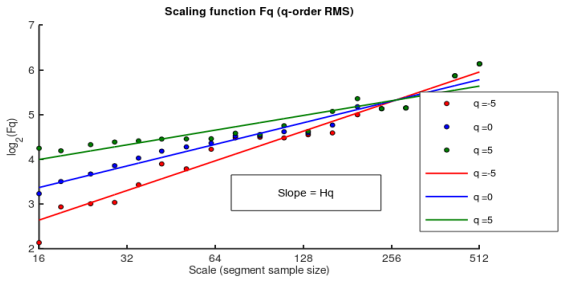

Figure 7 - Function Fq for the norm_avl

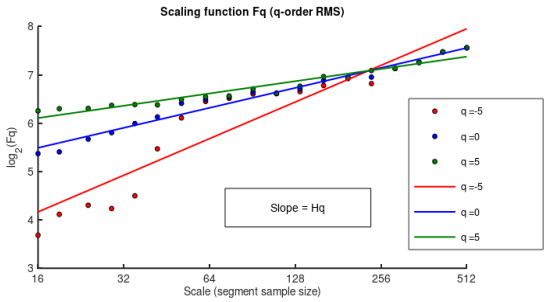

Figure 8 - Function Fq for the norm_avf

Figures9-14 show the q-order Hurst exponent Hq for the time series, where the colored dots represents the slopes Hq for q =−5, 0 and 5. Hq are q-dependent and time series are multifractal.

Figures 15-20 show mass exponent tq. The multifractal time series have mass exponents with a curved q-dependency and, consequently, a decreasing singularity exponent hq.

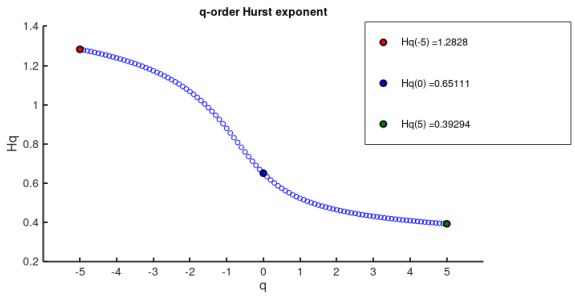

Figure 9 - Hurst exponent for the norm_1

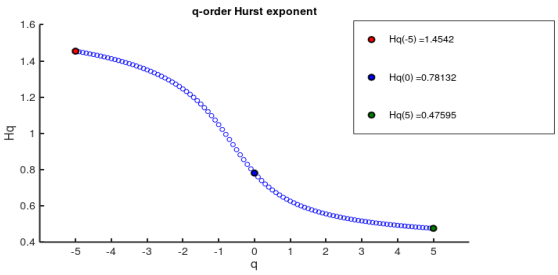

Figure 10 - Hurst exponent for the norm_2

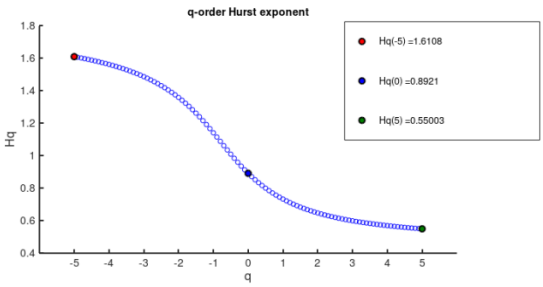

Figure 11 - Hurst exponent for the norm_3

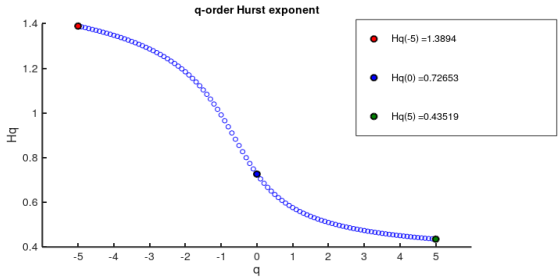

Figure 12 - Hurst exponent for the norm_avr

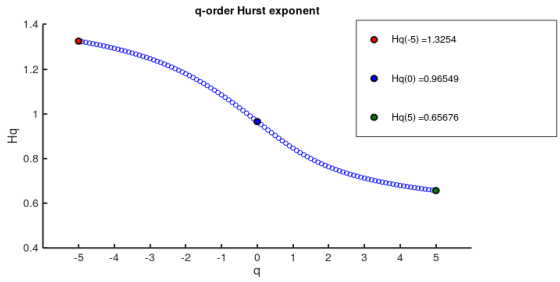

Figure 13 - Hurst exponent for the norm_avl

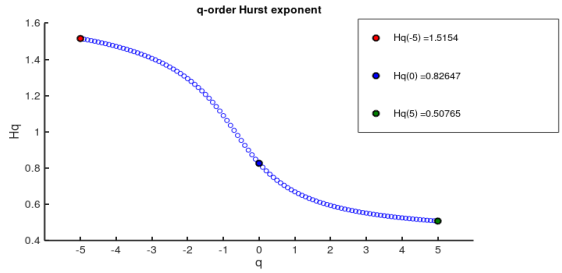

Figure 14 - Hurst exponent for the norm_avf

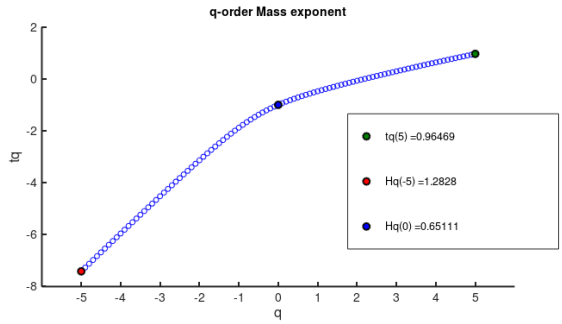

Figure 15 - Mass exponent for the norm_1

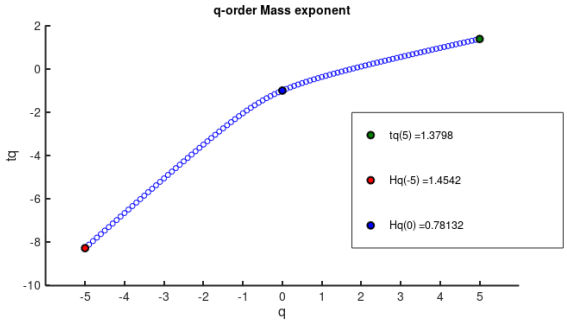

Figure 16 - Mass exponent for the norm_2

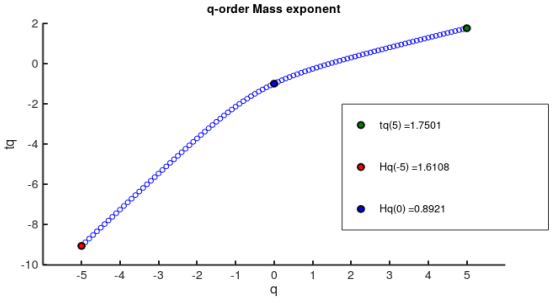

Figure 17 - Mass exponent for the norm_3

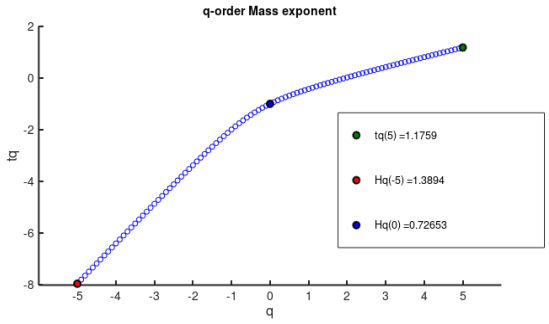

Figure 18 - Mass exponent for the norm_avr

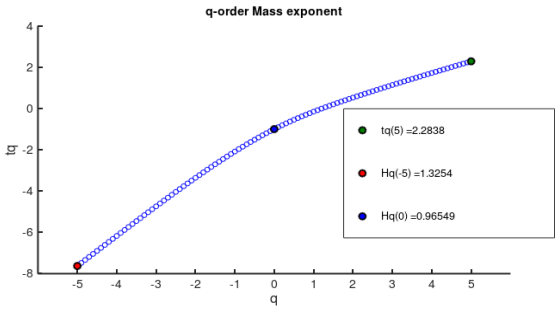

Figure 19 - Mass exponent for the norm_avl

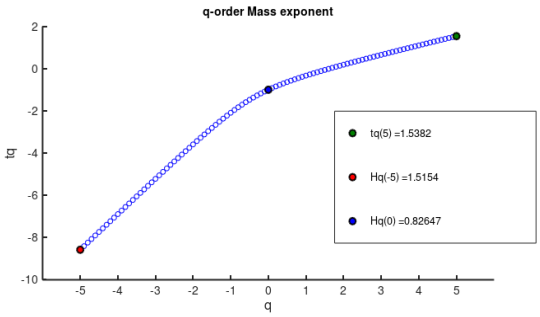

Figure 20 - Mass exponent for the norm_avf

The resulting spectrums are a large arc where the difference between the maximum and minimum hq are called the multifractal spectrum. Figures 21-26 show the resulting spectrumsand $D_q = f(\alpha)$, $h_q = \alpha$.

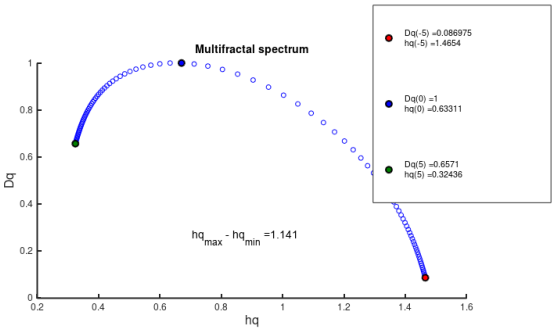

Figure 21 - Multifractal spectrum for the norm_1

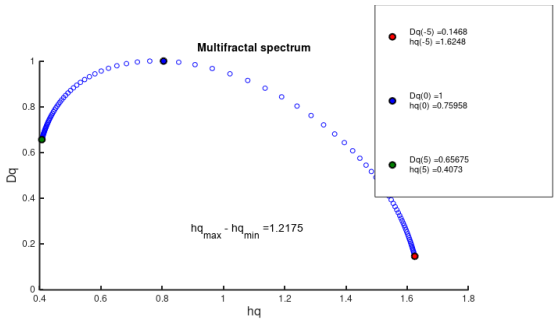

Figure 22 - Multifractal spectrum for the norm_2

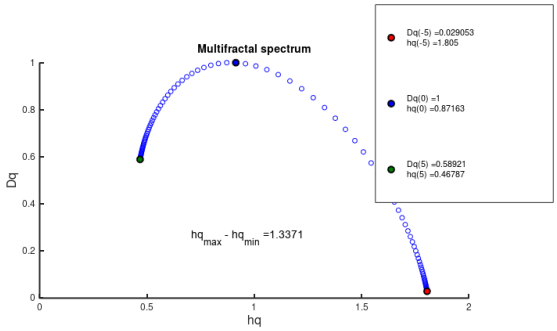

Figure 23 - Multifractal spectrum for the norm_3

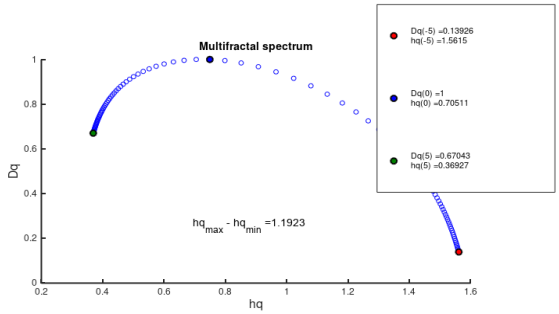

Figure 24 - Multifractal spectrum for the norm_avr

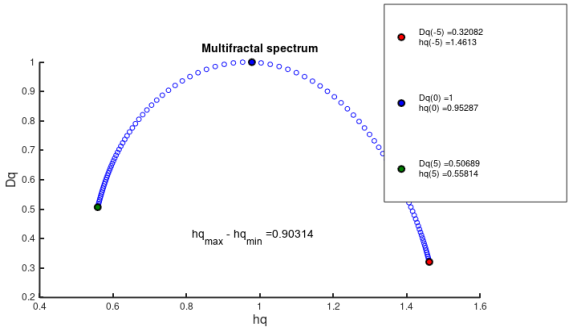

Figure 25 - Multifractal spectrum for the norm_avl

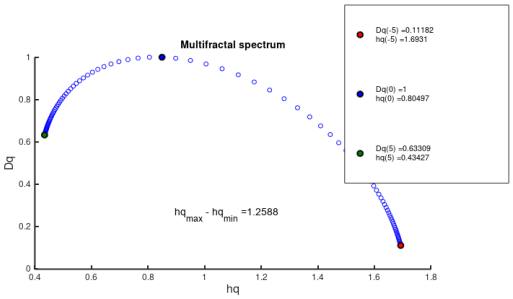

Figure 26 - Multifractal spectrum for the norm_avf

Conclusions

This work is to show the executed multifractal analysis of ECG data. The following results were obtained for all time series.

1. The scaling functions are q-dependent.
2. Hurst exponent Hq is q-dependent.
3. The time series have mass exponents with a curved q-dependency.
4. Time series have a wide range of multifractal spectrum.

For these reasons, the timeseries are multifractal.

Literature

1. Introduction to multifractal detrended fluctuation analysis in Matlab. //https://www.mathworks.com/matlabcentral/fileexchange/38262-multifractal-detrended-fluctuation-analyses
2. Программное обеспечение кардиографа. //https://vdd-pro.ru/ru/usb-kardiograf/programmnoe-obespechenie/
3. ECGControl. Руководство пользователя. //http://vdd-pro.ru/wp-content/uploads/2017/10/ECG_Control_User_Guide-1.pdf
4. Introductory by guide to identifying ECG irregularities. // https://www.dcbiomed.com/proimages/materials/EKG_Introduction/portableEKG_weitere_Informationen.pdf
5. Olsen L. Multifractal Analysis. A selected survey. //https://math.unt.edu/sites/www.math.unt.edu/files/users/cag0248/olsen-slides.pdf
6. Kantelhardt J.W., Zschiegner S.A., Koscielny-Bunde E. , Bunde A., Havlin S., Stanley H.E. Multifractal Detrended Fluctuation Analysis of Nonstationary Time Series. //https://arxiv.org/pdf/physics/0202070.pdf
7. Multifractal Analysis. // https://www.mathworks.com/help/wavelet/ug/multifractal-analysis.html

Бельков Д.В., Едемская Е.Н., Лазебная Л.А., Бойко В.Н. Мультифрактальный анализ детрендированных флуктуаций ЭКГ-данных. *Измерение электрических импульсов сердца с помощью электрокардиограммы (ЭКГ) стало основным методом выявления проблемных состояний сердца, таких как сердечные приступы или нарушение ритма сердца. Оно широко применяется в кардиологии для лучшего понимания динамического поведения нарушений сердечного ритма, особенно при «нерегулярном» сердцебиении, и может быть охарактеризовано как монофрактальное или мультифрактальное, а не просто случайное. Используя временные ряды данных электрокардиограммы, можно применять мультифрактальный анализ флуктуаций (MDFA) для исследования возможной мультифрактальной структуры данных ЭКГ. Данная работа посвящена тестированию мультифрактальной структуры временных рядов данных электрокардиограммы. Исходные данные получены с помощью программы ECGControl. Исследование выполнено в среде Octave.*

Ключевые слова: мультифрактальный анализ флуктуаций, монофрактальный, многофрактальный, временные диаграммы

Belkov D.V., Edemskaya E.N., Lazebnaya L.A, Boyko V.N. Multifractal detrended fluctuation analysis in Octave for ECG data. *Measuring the electrical impulses of the heart through an electrocardiogram (ECG) has become the primary method for detecting problematic heart conditions such as heart attacks or cardiac arrhythmia. Gaining more insight into the dynamical behavior of heartbeat irregularities would have meaningful applications in cardiology, especially if an "irregular" heartbeat could be characterized as being monofractal or multifractal, as opposed to being merely random. Using times series from electrocardiogram data, anyone is able to apply multifractal detrended fluctuation analysis (MDFA) to investigate possible multifractal structure of ECG data. Our work is aimed to test multifractal structure in time series data from an electrocardiogram. We examine data from ECGControl program. The research is executed in Octave runtime.*

Key words: *MDFanaly, multifractal, monofractal, random, time series.*